the minima of the neutral stability curves. These critical values for $Pr = 0.7$ and 7 are listed in Table 1. For $Pr = 7$ and $f_w = 0$, the critical Grashof number and wave number are computed to be 57.94 and 0.96, respectively, as compared to 56.326 and 0.942 reported in Lee et al.[8] appropriate for an impermeable surface. The results indicate that suction tends to stabilize the flow, while blowing tends to destabilize it. The reason that suction gives rise to a more stable flow is due to the fact that the effect of suction is to suck away the warm fluid on the plate and suppress the occurrence of vortices, and consequently suction stabilizes the vortex mode of instability. It is also seen that the critical wave number k^* increases as the flow changes from strong blowing to strong suction (i.e., f_w increases).

Conclusions

A dimensionless blowing or suction parameter $f_w = (-A/3\upsilon)[g\beta(T_w - T_\infty)/5\upsilon^2]^{-1/5}$ has been successfully employed in this note to investigate the effects of surface blowing and suction on the flow and vortex instability of a horizontal laminar natural convection boundary-layer flow. The numerical results demonstrate that blowing ($f_w < 0$) reduces the heat transfer rate and destabilizes the flow as compared with the case of an impermeable surface, while for suction ($f_w > 0$) the opposite trend is true.

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Mixed Convective Heating of a Moving Plate in a Parallel Duct

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Nomenclature

d = dimensionless thickness of moving plate, d^+/L_h
Gr = Grashof number, $g\beta(T_h - T_i)L^3/(\nu_f)^2$
g = gravitational acceleration
H_l = height of lower subchannel
H_u = height of upper subchannel
k = thermal conductivity
k^* = thermal conductivity ratio, k_p/k_f
L_{ds} = downstream duct length
L_{us} = upstream duct length
Nu = Nusselt number
Pe_p = plate Peclet number, u_pL_h/α_p
Pr = Prandtl number, ν_f/α_f
Re = Reynolds number, u_pL_h/ν_f
T = temperature
u_p = speed of moving plate
x, y = Cartesian coordinates
α = thermal diffusivity
β = thermal expansion coefficient
θ = dimensionless temperature, $(T - T_i)/(T_h - T_i)$
ν = kinematic viscosity
ϕ = inclination angle
Ψ = stream function
ω = vorticity

Subscripts

b = bottom plate surface or heater
f = fluid
h = heater
l = lower subchannel
p = moving plate
t = top plate surface or heater
u = upper subchannel

Introduction

VERY often heating or cooling of a moving surface can be encountered in various manufacturing processes such as the processes of rolling, extrusion, and continuous casting; drying and curing processes; and other thermal treatments of materials. After the pioneering investigations by Sakiadis[1] on the analysis of the boundary-layer flow induced on a continuous moving sheet or cylinder, heat and momentum transport phenomena due to a moving surface have been extensively studied for the past few decades. Comprehensive reviews on this subject can be found in recent works,[2–4] and therefore there is no need to repeat them here. It is noted that the previous works primarily focused on the boundary-layer cooling process of a moving surface in a quiescent ambient fluid of unconfined domain. In practice, however, the moving surface may be confined in a finite fluid space restricted by the neighboring solid boundaries. Moreover, the convection transport process induced by the moving surface within a confined domain can be expected to be more complex than that in an unconfined space. Therefore, an understanding of the convection heat transport phenomenon induced by a mov-

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ing surface in a restricted fluid space is of both practical and theoretical importance.

Problem Formulation and Numerical Method

This Note considers a two-dimensional laminar mixed convective heating process of a moving plate of thickness d^+ inside a parallel duct. An isothermally heated section at T_h of length L_h is, respectively, situated at the top and bottom wall of the duct symmetrically, and other than the heating sections, the top and bottom duct walls are assumed to be insulated thermally. The fluid flow in the duct may be due to combined causes of viscous and buoyancy effects. In addition, the interaction of conduction heat transfer inside the moving plate with the mixed convection of fluid within the duct is taken into consideration in the present study. At the entrance of the duct, the fluid flow is assumed hydrodynamically fully developed and temperatures of the fluid as well as the moving plate are taken as uniform at T_i. The fluid is considered to be Newtonian adhering to the Boussinesq approximation. Moreover, the moving plate is considered to have a constant thermal conductivity. The appropriate governing equations in terms of stream function, vorticity, and temperature for the problem considered can be nondimensionalized to obtain

For the fluid

$$\frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y} = \frac{1}{Re}\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) + \frac{Gr}{Re^2}\left(\cos\phi \frac{\partial \theta}{\partial x} - \sin\phi \frac{\partial \theta}{\partial y}\right) \tag{1}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \tag{2}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} = \frac{1}{RePr}\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) \tag{3}$$

For the moving plate

$$\frac{\partial \theta}{\partial x} = \frac{1}{Pe_p}\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) \tag{4}$$

The boundary conditions for the problem are as follows:

$$y = 0, \text{ or } H; \quad \psi = 0, \quad \frac{\partial \theta}{\partial y} = 0$$

$$\text{for } -L_{us} \le x \le 0 \text{ or } 1 \le x \le (1 + L_{ds})$$

$$\theta = 1 \quad \text{for} \quad 0 \le x \le 1 \tag{5a}$$

$$x = -L_{us}; \quad \theta = 0, \quad \psi = \frac{y^2}{2H_l} \quad \text{for} \quad 0 \le y \le H_l$$

$$\psi = \frac{-1}{2H_u}(H - y)^2 \quad \text{for} \quad (H - H_u) \le y \le H \tag{5b}$$

$$x = L_{ds}; \quad \frac{\partial \omega}{\partial x} = 0, \quad \frac{\partial^2 \psi}{\partial x^2} = 0, \quad \frac{\partial^2 \theta}{\partial x^2} = 0 \tag{5c}$$

The outflow boundary condition, Eq. (5c), adopted here are a result of extensive numerical experimentation with several types of possible outflow boundary conditions.

The set of dimensionless governing differential equations, Eqs. (1–4), was discretized in finite difference form. The numerical scheme adopted here basically follows that described in an earlier study.[5] To select appropriate computational domain for the present problem, computations were conducted with several dimensionless lengths of the duct, $L_d(=L_{us} + 1 + L_{ds})$, to ensure that the field variables at the inflow and outflow boundaries were largely insensitive to the recirculations around the heated section of the duct; and the values of $L_{us} = 8$ and $L_{ds} = 40$ were selected for the present calculations. Moreover, a composite grid system was constructed consisting of three mesh zones: a uniform mesh over the heated section and an exponentially nonuniform mesh respectively for the upstream and downstream regions. Based on a series of grid size test, the results were mainly generated using a mesh of 105 (x direction) by 73 (y direction) as a result of compromise of accuracy and cost. Furthermore, of the 73 y direction grid points, 5 grid points were laid uniformly over the moving plate.

Results and Discussion

Numerical results have been generated for air as the heat transfer fluid ($Pr = 0.7$) inside a duct of $H_l = H_u = 1$ and $d = 0.1$ with other parameters in the ranges: $Re = 0.1–400$, $Gr = 0–10^4$, $Pe_p = 0.14$, 7, and 70, $k^* = 8–50$, and $\phi = 0–90$, deg. In the following, results will be presented to illustrate the effects of Gr, Re, and ϕ on the mixed convective heating process under consideration.

In Fig. 1, the effect of Gr on the local Nusselt number distributions on the heaters, as well as on the surfaces of the moving plate, is conveyed. As Gr is increased, the local Nusselt number adjacent to the leading edge of the top heater tends to decrease due to the diminishing effect of forced convection; while in the remaining portion of the top heater, the local Nusselt number is seen to be slightly increased as a result of the buoyancy effects. Consequently, a rather uniform distribution of heat flux on the top heater is observed at $Gr = 10^4$. On the other hand, a trend generally opposite to the foregoing on the top heater is obtained for the heat flux distribution along the bottom heater. For $Gr \le 10^2$, the local Nusselt number on the bottom heater remains largely unaffected as displayed in Fig. 1. As Gr is increased up to 10^4, the local Nusselt number profile appears to be drastically changed, signifying the onset of the buoyancy effects. A great increase of heat transfer rate occurs over the front half of the bottom heater and a minimum Nusselt number arises at $x \approx 0.8$ where the thermal plume activity is detected.

Moreover, along the top surface of the moving plate, as Gr increases, the peak of the local Nusselt number shown in Fig. 1 is gradually decreased in magnitude and is shifted upstream at $Gr = 10^4$, resulting in a local minimum value at a location around which the counter-rotating buoyant recirculations meet. As a result, the energy inflow to the top surface of moving plate becomes less localized, spreading out particularly in the upstream direction owing to the stronger counterclockwise recirculation in the region. As for the bottom surface of the moving plate, the main features of the local Nusselt number distribution appear to be somewhat unchanged for $Gr < 10^2$. Further increase of Gr up to 10^4, as displayed in Fig. 1, leads to a more pronounced localized heat flux profile over the bottom surface of the plate. Above all, as indicated by the foregoing, the convective heating behavior on the top and bottom surfaces of the moving plate within the duct is affected differently by the buoyancy effects.

Next, the effect of Reynolds number is considered. As exemplified in Fig. 2, the local Nusselt number profile for the heaters exhibits a substantial increase as Re is increased beyond 10. The local heat inflow on the moving plate substantially declines with increase of Re owing to its lower heat capacity. Furthermore, the downstream shift of the peak Nusselt numbers on the surfaces of the plate with increasing Re clearly reflects the effect of shear-driven convection.

Finally, attention will be turned to the influence of inclination angle of the duct ϕ on the mixed convection heating process considered. Figure 3 exemplifies the results of isotherms, streamlines, and longitudinal velocity profiles for three values of $\phi(=30, 60, 90 \text{ deg})$ at $Gr = 10^3$, $Re = 1$, $k^* = 8$, and $Pe_p = 7$. By progressively tilting the duct toward the gravitationally vertical orientation ($\phi = 90$ deg), as expected, the buoyancy force acts increasingly as an aiding effect for

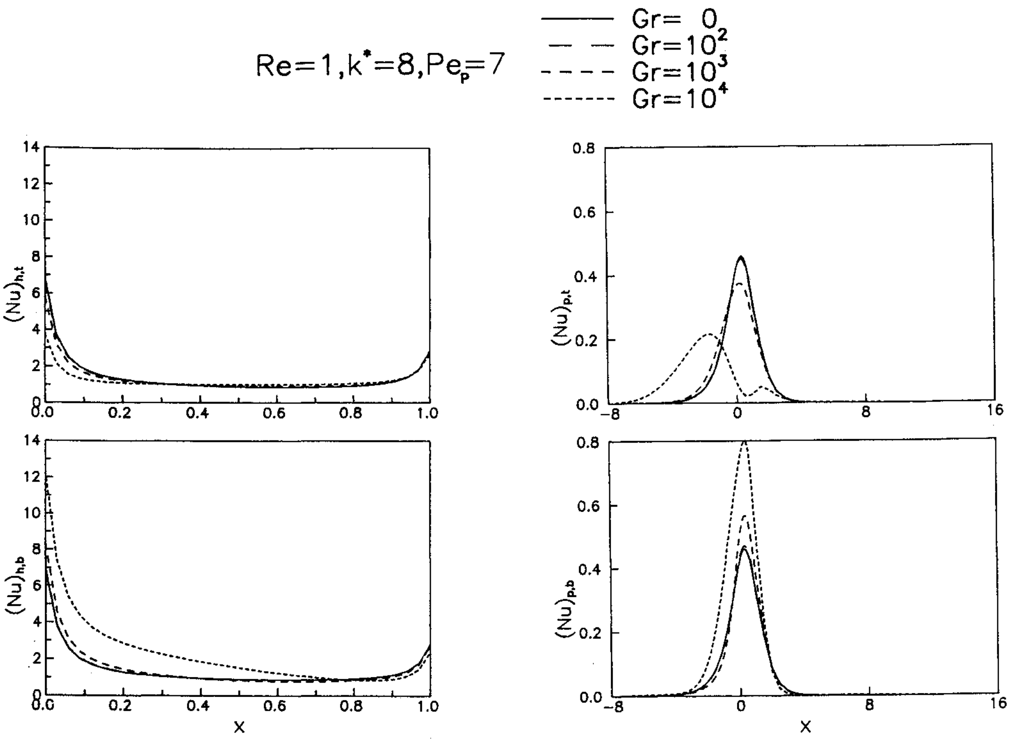

Fig. 1 Buoyancy effect on local heat transfer distributions.

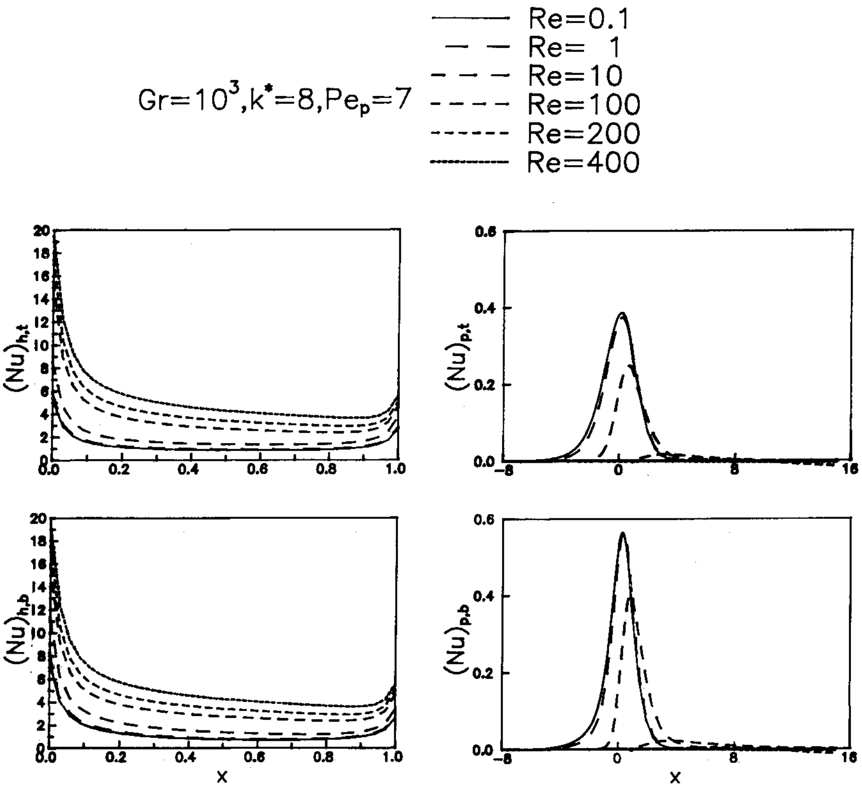

Fig. 2 Variations of local mixed convection Nusselt number with *Re*.

the shear-driven forced convection. From Fig. 3 it can be noted that the counterclockwise buoyant recirculation in the upper subchannel is gradually impeded with increasing inclination angle and eventually disappears; meanwhile a clockwise recirculation arises downstream of the top heater at $\phi = 30$ deg, and becomes the dominant flow structure in the upper subchannel at $\phi = 60$ deg. At $\phi = 90$ deg, the flowfields in both subchannels exhibit a symmetry with respect to the moving plate as expected, and are characterized with a recirculating core surrounded by a relatively slow, thin upflow driven by the moving plate within each subchannel as indicated by the velocity profiles in Fig. 3c. Furthermore, the local heat flux distributions on the thermally active surfaces (heater and plate surface) in the upper subchannel (not shown) are found to be more sensitive to varying inclination than those in the lower subchannel.

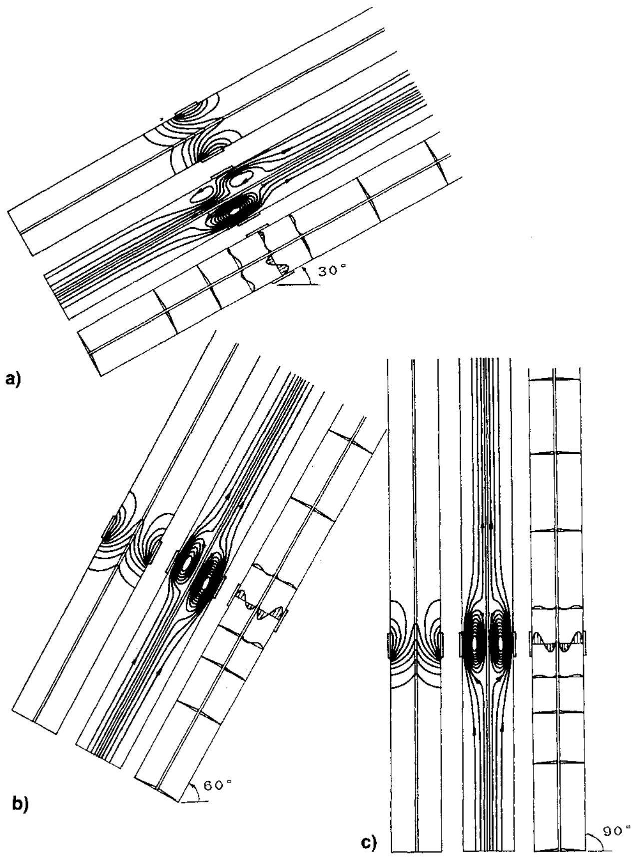

Fig. 3 Isotherms, streamlines, and velocity profiles for inclined duct at $Re = 1$, $Gr = 10^3$, $Pe_p = 7$, and $k^* = 8$: a) $\psi_{max} = 1.5587$, $\psi_{min} = -0.6683$, $\Delta\psi = 0.13$; b) $\psi_{max} = 1.7000$, $\psi_{min} = -1.1795$, $\Delta\psi = 0.13$; and c) $\psi_{max} = 1.5606$, $\psi_{min} = -1.5606$, $\Delta\psi = 0.13$.

Concluding Remarks

In this study, numerical simulations by means of a finite difference method have been performed to render the effects of the relevant physical parameters of the problem considered (Re, Gr, and ϕ) on the interaction between the shear-driven flow and the buoyant recirculating flow during the heating process of the moving plate. Results indicate that the shear-driven flowfields in the divided subchannels can be strongly affected by the buoyancy force due to an increase of Grashof number; a bicellular recirculation arises respectively within the subchannels in the region around the heaters. Moreover, an increase in Grashof number tends to promote the upstream diffusion effect due to the buoyant recirculation, particularly for the upper channel. Accordingly, the local Nusselt number on the top surface of the moving plate becomes less localized shifting in the upstream direction. Moreover, the flowfield and temperature distribution in the upper subchannel are found to be more sensitive to the inclination of the duct. For the range of inclination considered, the temperature as well as the heat transfer on the moving plate are rather unaffected by the variation of the duct orientation.

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Bias Error Reduction Using Ratios to Baseline Experiments—Heat Transfer Case Study

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Introduction

PRECISION uncertainties can be reduced by repeated trials and averaging. Bias or fixed uncertainties cannot be so reduced. In many engineering experimental programs, the ultimate result can practically be expressed as a ratio of two experimentally determined quantities. If experiments are conducted in a nominally identical manner using the same instruments, many of the bias uncertainties in the two experiments will be strongly correlated. When the results are presented as a ratio, these correlated bias uncertainties will tend to cancel. For bias-dominated experiments, the ratio can be considerably more certain than the individual measurements.

This scheme for bias error reduction can give considerable advantage when parametric effects are being investigated experimentally. As an example, consider a set of experiments where the effect of surface finish (riblets) on convective heat transfer is being studied. First, baseline experiments would be conducted with smooth surfaces. Then, a series of experiments would be conducted with different surface finishes, and a comparison of the heat transfer on each with the smooth baseline case would be made. The desired change in result is small and often on the same order as the uncertainties in the measurements. If the raw heat transfer data are compared directly, the small changes in results will be colored by the

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